SEARCH METHODS IN AI

HEURISTIC SEARCH METHODS

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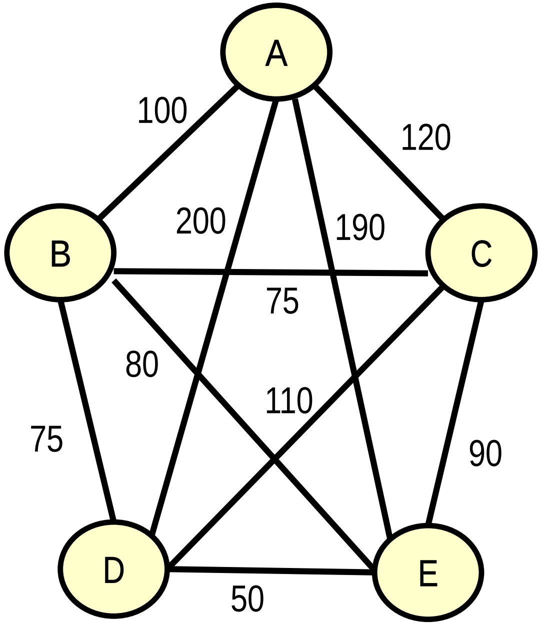
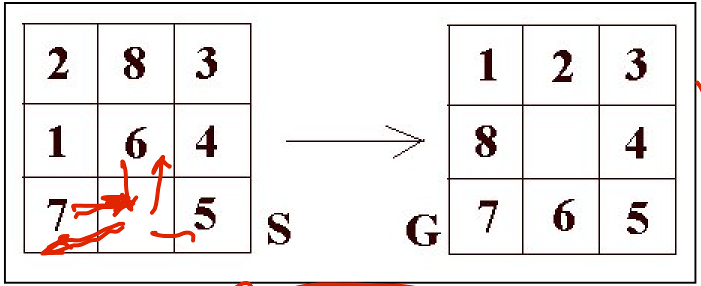
BASICS OF STATE SPACE MODELLING

- STATE or CONFIGURATION:
  - A set of variables which define a state or configuration
  - Domains for every variable and constraints among variables to define a valid configuration
- STATE TRANSFORMATION RULES or MOVES:
  - A set of RULES which define which are the valid set of NEXT STATE of a given State
  - It also indicates who can make these Moves (OR Nodes, AND nodes, etc)
- STATE SPACE or IMPLICIT GRAPH
  - The Complete Graph produced out of the State Transformation Rules.
  - Typically too large to store. Could be Infinite.
- INITIAL or START STATE(s), GOAL STATE(s)
- SOLUTION(s), COSTS
  - Depending on the problem formulation, it can be a PATH from Start to Goal or a Sub-graph of And-ed Nodes
- SEARCH ALGORITHMS
  - Intelligently explore the Implicit Graph or State Space by examining only a small sub-set to find the solution
  - To use Domain Knowledge or HEURISTICS to try and reach Goals faster

EXAMPLE PROBLEMS



Travelling Salesperson



Sliding Puzzle

shortest path

Knapsack Packing

which subset should I take so that we get maximum value within the weight limits

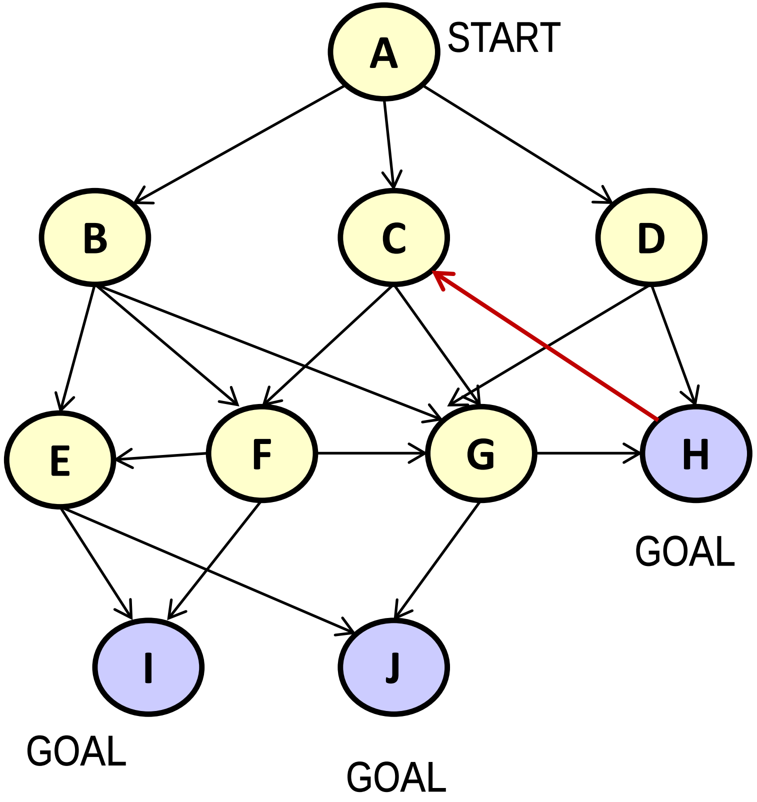
MAXIMIZATION

SEARCHING IMPLICIT GRAPHS

The various Search Algorithms include

- BASIC Algorithms: Depth-First (DFS), Breadth-first (BFS), Iterative Deepening (IDS)
- COST-based Algorithms: Depth-First Branch-and-Bound, Best First Search, Best-First Iterative Deepening
- Widely Used Algorithms: A* and IDA* (Or Graphs), AO* (And/Or Graphs), Alpha-beta Pruning (Game-Trees)

EXAMPLE: SEARCHING A STATE SPACE GRAPH



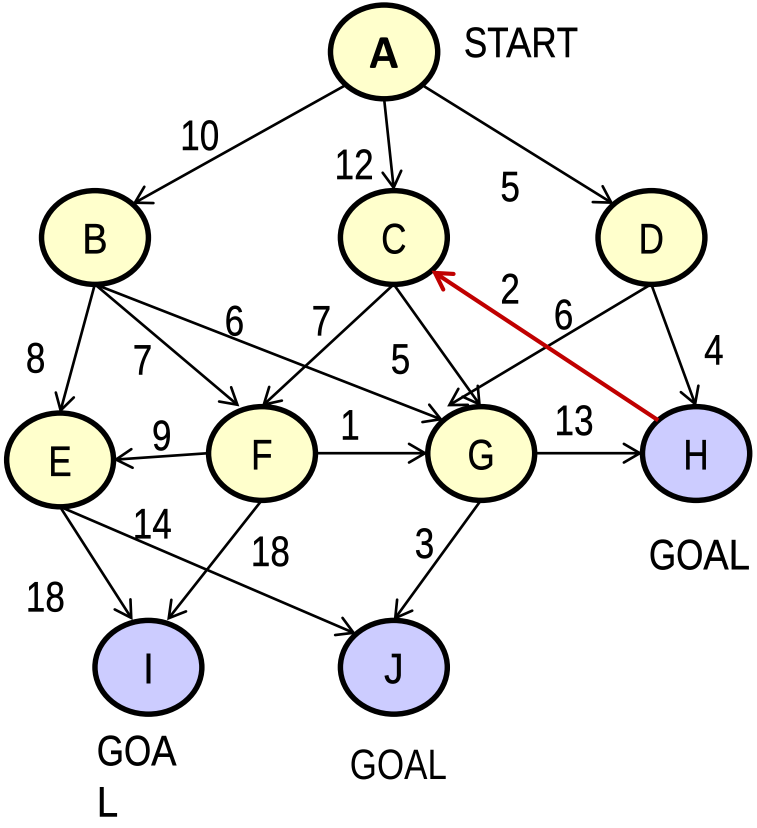
- DEPTH-FIRST SEARCH (DFS)
- BREADTH-FIRST SEARCH (BFS)
- ITERATIVE DEEPENDING SEARCH (IDS)
- PROPERTIES
    - SOLUTION GUARANTEES
    - MEMORY REQUIREMENTS

BASIC ALGORITHMS:DFS, IDS, BFS

1. [Initialize] Initially the OPEN List contains the Start Node s. CLOSED List is Empty.
2. [Select] Select the first Node n on the OPEN List. If OPEN is empty, Terminate
3. [Goal Test] If n is Goal, then decide on Termination or Continuation / Cost Updation
4. [Expand]
   a) Generate the successors n_1, n_2, …. n_k, of node n, based on the State Transformation Rules
   b) Put n in LIST CLOSED
   c) For each n_i, not already in OPEN or CLOSED List, put n_i in the FRONT (for DFS) / END (for BFS) of OPEN List
   d) For each n_i already in OPEN or CLOSED decide based on cost of the paths
5. [Continue] Go to Step 2

Algorithm IDS Performs DFS Level by Level Iteratively (DFS (1), DFS (2), ……. and so on)

SEARCHING STATE SPACE GRAPHS WITH EDGE COSTS



- COST ORDERED SEARCH:
  - DFBB
  - Best First Search, → ordered by g(n) values
  - Best First IDS
  - Use of HEURISTIC Estimates: Algorithm A* (Or Graphs), AO* (And/Or Graphs)
- PROPERTIES
  - SOLUTION GUARANTEES
  - MEMORY REQUIREMENTS

g(n) = cost/current cost of the node from start

HEURISTIC SEARCH

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- SOLUTION(s), COSTS
  - Depending on the problem formulation, it can be a PATH from Start to Goal or a Sub-graph of And-ed Nodes
- HEURISTICS
  - Estimates of cost from a given state to goal. This, along with the current cost of the path from start till now is used to guide the search
- HEURISTIC SEARCH ALGORITHMS
  - Algorithm A*, Depth-First Branch & Bound, IDA*, AO*, Alpha-Beta, etc
  - Knowledge vs Search

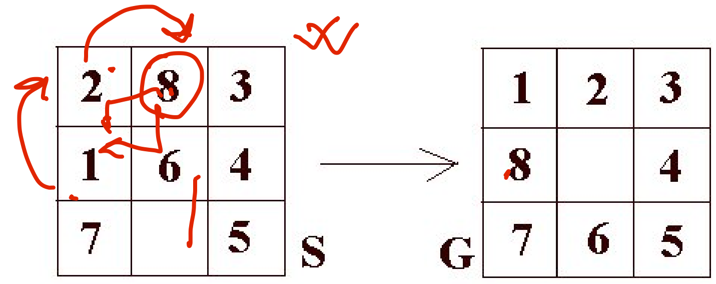
$g(n)$

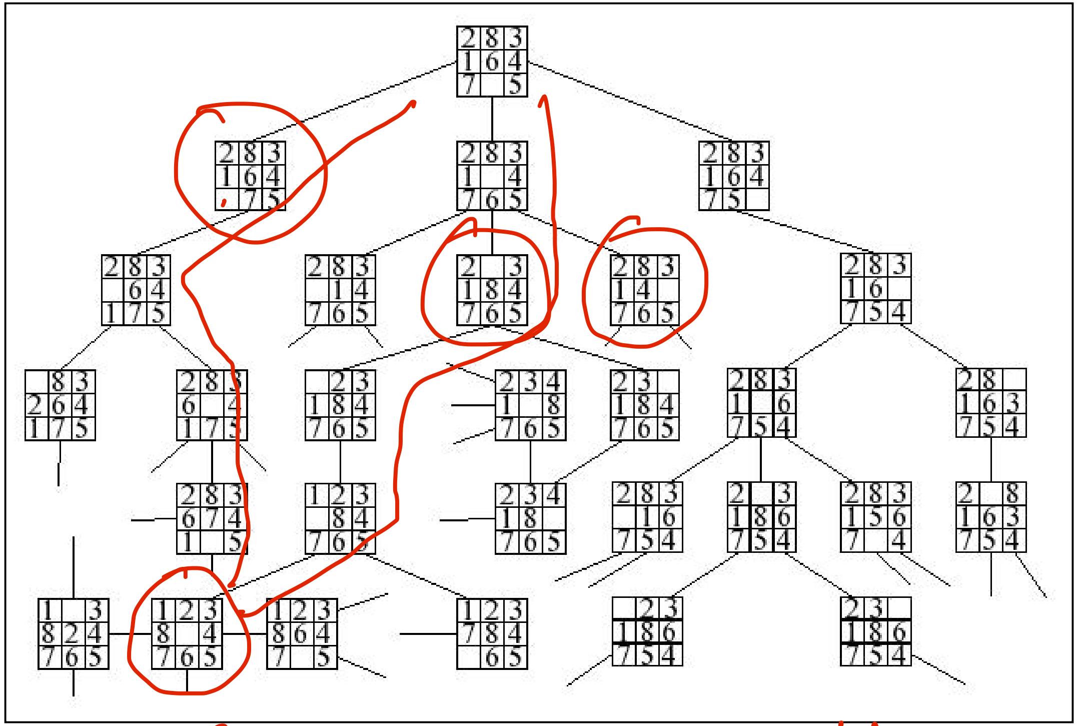
$h(n)$ = estimated cost of the best path from n to goal

heuristic estimate

$$f(n) = g(n) + h(n)$$

EXAMPLE OF HEURISTICS: 8 PUZZLE PROBLEM





HEURISTIC 1: NUMBER OF MISPLACED TILES

HEURISTIC 2: SUM OF TILE DISTANCES FROM GOAL

h(n) = estimated cost of the best path from n to goal.

4 misplaced tiles. lower bound

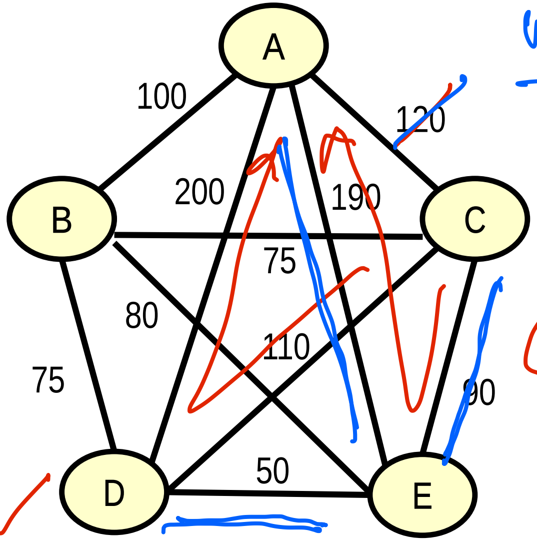
1 + 1 + 0 + 0 + 0 + 1 + 0 + 2 = 5

Both are underestimates

h2 is more accurate than h1

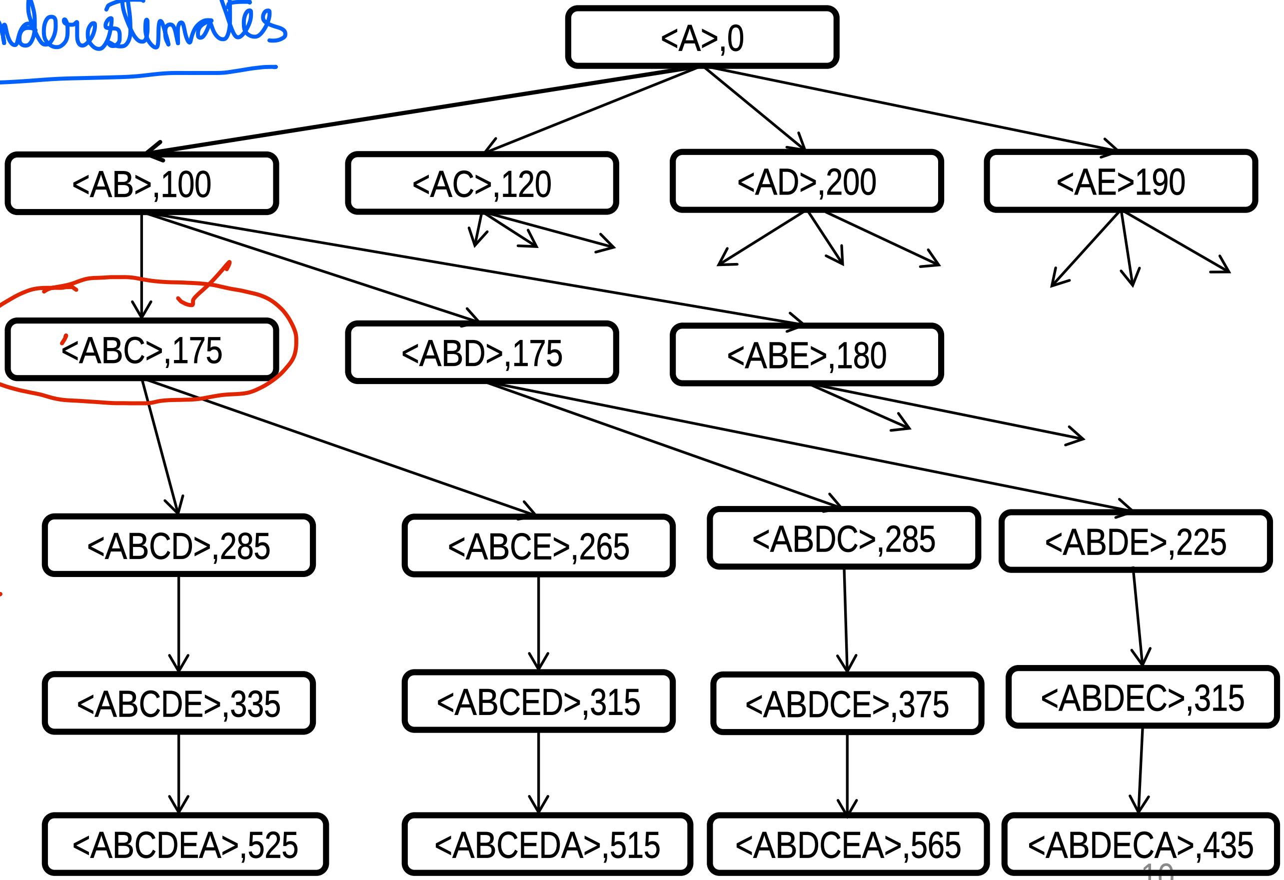
TRAVELLING SALESPERSON PROBLEM

underestimates

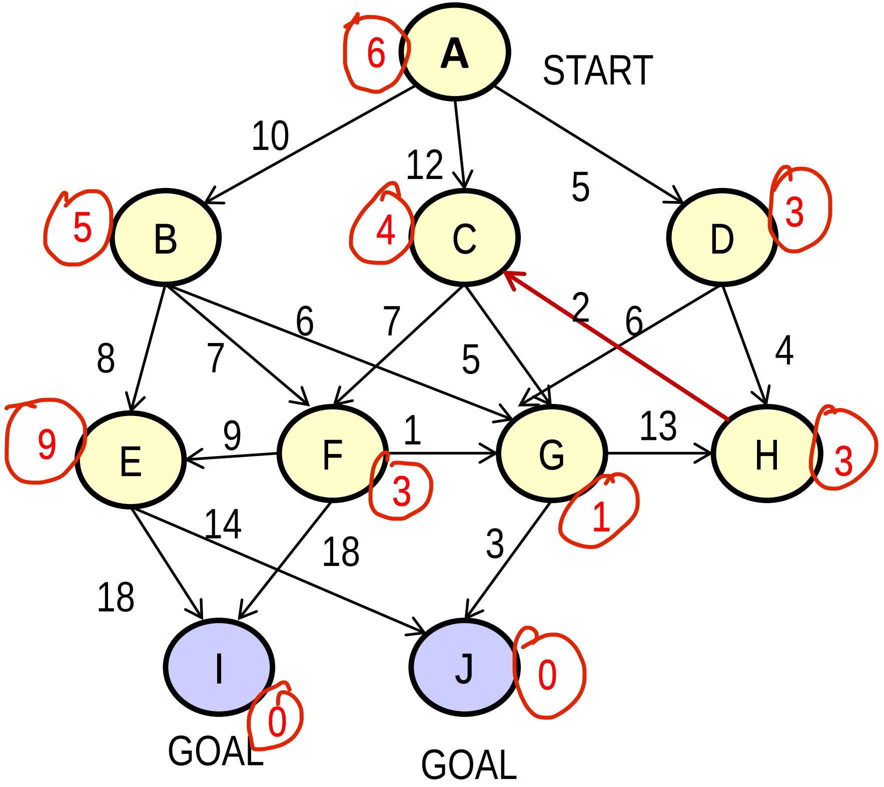


HEURISTIC 1: COST OF SHORTEST PATH FROM CURRENT NODE TO START THROUGH REMAINING NODES ONLY

HEURISTIC 2: COST OF MINIMUM COST SPANNING TREE OF REMAINING NODES



SEARCHING STATE SPACES WITH EDGE COSTS, HEURISTIC ESTIMATES



- HEURISTIC SEARCH ALGORITHMS:
  - DFBB
  - A*: Best First Search,
  - IDA*: Iterative Deepening A*
  - Every edge (n, m) in the graph has a cost c(n,m) > 0.
  - **HEURISTIC Estimates: h(n) ≥ 0 at** every node is the estimated cost of the minimum cost path from node n to goal

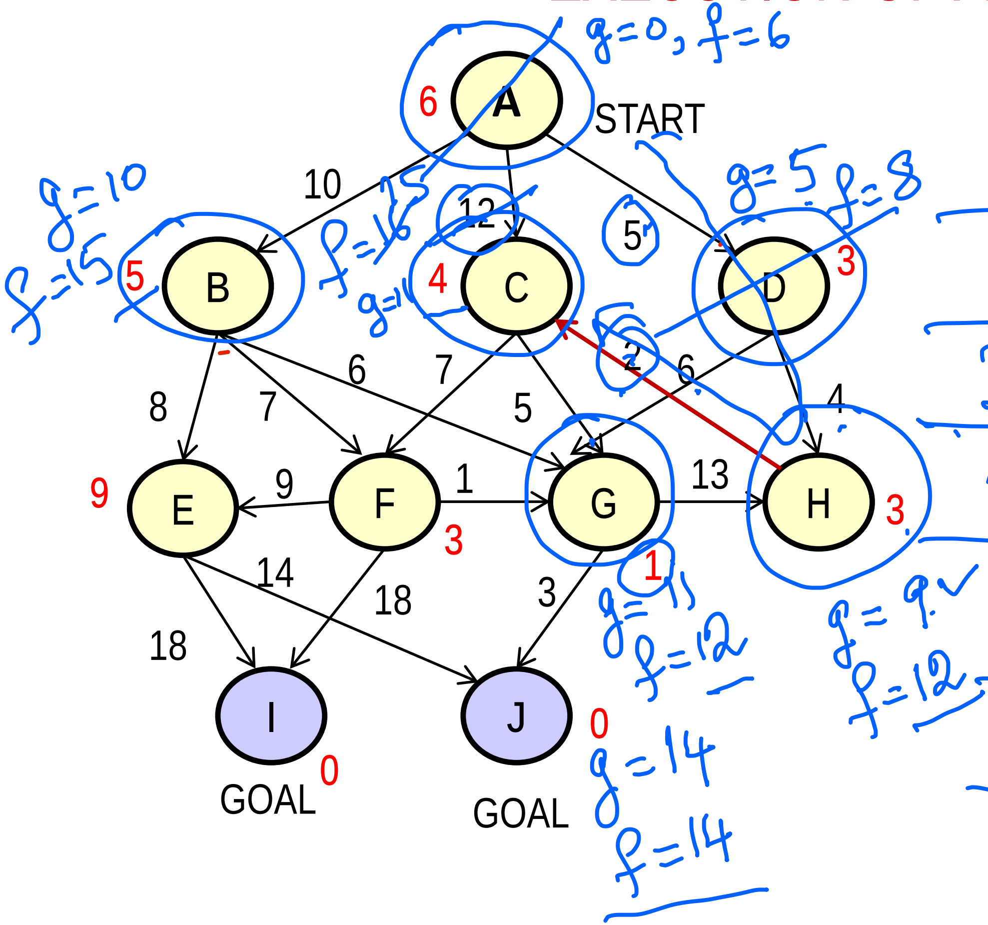
- PROPERTIES
  - SOLUTION GUARANTEES
  - MEMORY REQUIREMENTS

ALGORITHM A* (BEST FIRST SEARCH IN OR GRAPHS)

Each Node n in the algorithm has a cost g(n) and a heuristic estimate h(n), f(n) = g(n + h(n).
Assume all c(n,m) > 0

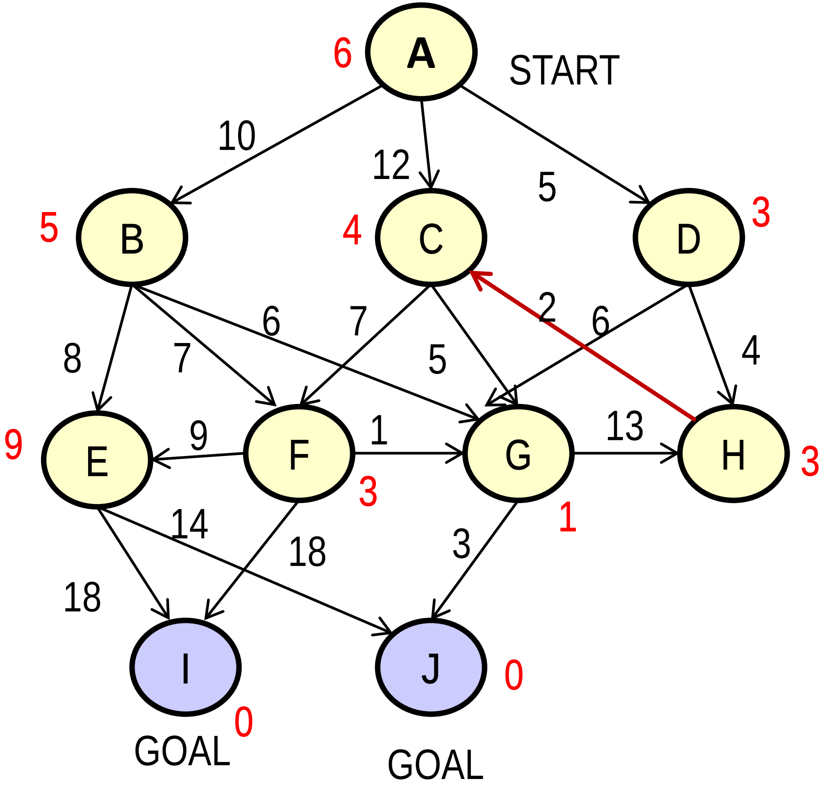
1. [Initialize] Initially the OPEN List contains the Start Node s. g(s) = 0, f(s) = h(s). CLOSED List is Empty.
2. [Select] Select the Node n on the OPEN List with minimum f(n). If OPEN is empty, Terminate with Failure
3. [Goal Test, Terminate] If n is Goal, then Terminate with Success and path from s to n.
4. [Expand]
   a) Generate the successors n_1, n_2, …. n_k, of node n, based on the State Transformation Rules
   b) Put n in LIST CLOSED
   c) For each n_i, not already in OPEN or CLOSED List, compute
      a) g(n_i) = g(n) + c(n, n_i), f(n_i) = g(n_i) + h(n_i) , Put n_i in the OPEN List
   d) For each n_i already in OPEN, if g(n_i) > g(n) + c(n,n_i), then revise costs as:
      a) g(n_i) = g(n) + c(n, n_i), f(n_i) = g(n_i) + h(n_i)
5. [Continue] Go to Step 2

EXECUTION OF ALGORITHM A*



	OPEN	CLOSED
1	A	
2	B, C, D	A
3	B, C (16), G, H	D
4	B, C (15), G	H
5	B (15), C (15), J (14)	G
		J

EXECUTION OF ALGORITHM A*



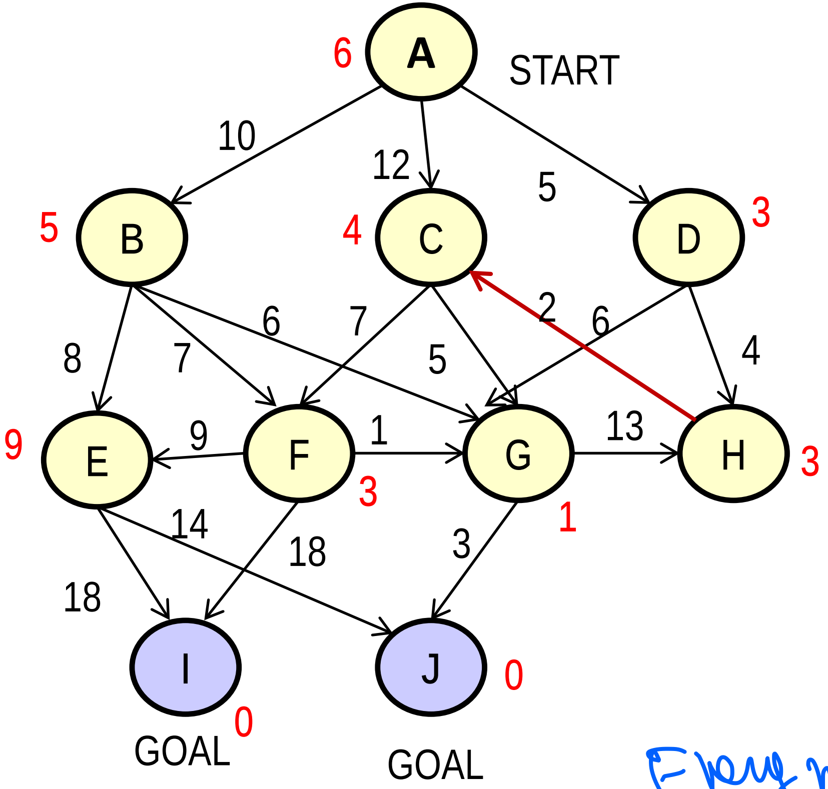
1. OPEN = {A[0,6,6]}, CLOSED = {}
2. OPEN = {B[10,5,15,A], C[12,4,16,A], D[5,3,8,A]}, CLOSED = {A}
3. OPEN = {B[10,5,15,A], C[12,4,16,A], G[11,1,12,D], H[9,3,12,D]}, CLOSED = {A,D}
4. OPEN = {B[10,5,15,A], C[11,4,15,H], G[11,1,12,D]}, CLOSED = {A,D,H}
5. OPEN = {B[10,5,15,A], C[11,4,15,H], J[14,0,14,G]}, CLOSED = {A[],D[A],H[D],G[D]}
6. Goal J found. Terminate with cost 14 and path A,D,G,J.

Best-first Algorithm

First solution is the one we terminate with

PROPERTIES OF ALGORITHM A*

Heuristics are assumed to be MONOTONIC



IF HEURISTIC ESTIMATES ARE NON-NEGATIVE LOWER BOUNDS AND EDGE COSTS ARE POSITIVE:

- FIRST SOLUTION IS OPTIMAL
- NO NODE IN CLOSED IN EVER REOPENED
- WHENEVER A NODE IS REMOVED FROM OPEN ITS MINIMUM COST FROM START IS FOUND
- EVERY NODE n WITH f(n) LESS THAN OPTIMAL COST IS EXPANDED
- IF HEURISTICS ARE MORE ACCURATE THEN SEARCH IS LESS

Never come to a node in CLOSED with lower cost

Every node $f(n) <$ optimal cost is expanded

$h_1 \leq h_2 \rightarrow$ more accurate

ALGORITHM DFBB

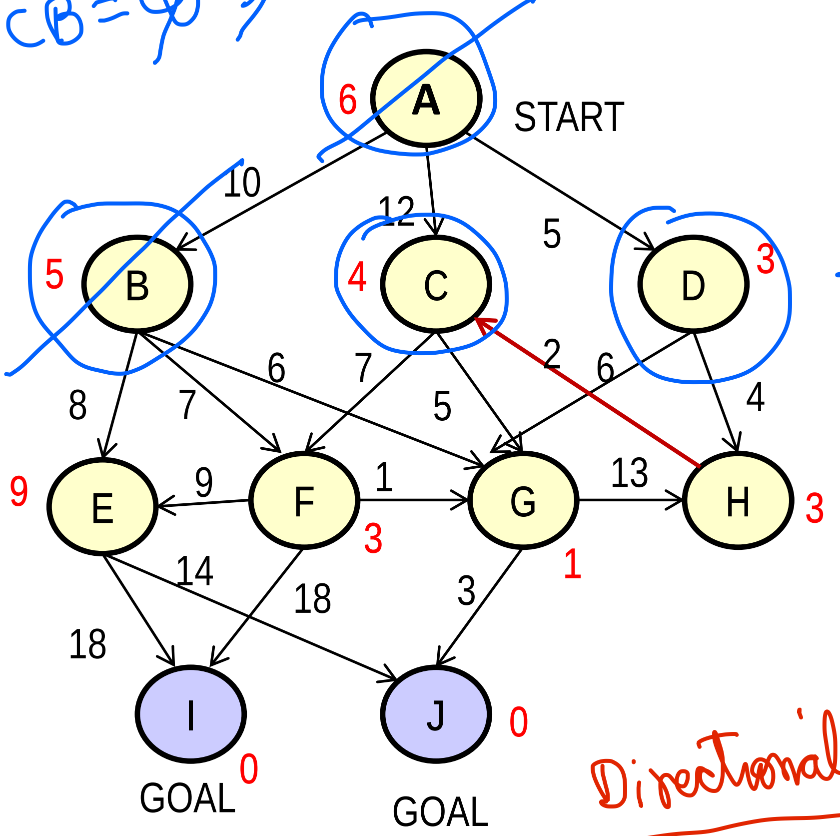
DEPTH FIRST BRANCH AND BOUND (DFBB)

1. Initialize Best-Cost to INFINITY
2. Perform DFS with costs and Backtrack from any node n **whose f(n) ≥ Best**-Cost
3. On reaching a Goal Node, update Best-Cost to the current best
4. Continue till OPEN becomes empty

PRUNING

CB = ∞ 36 32 20

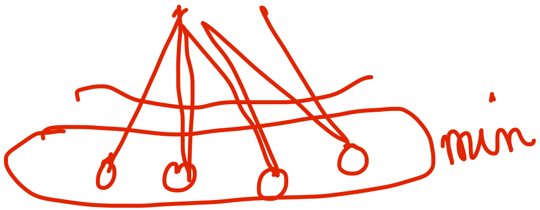
EXECUTION OF DFBB



Directional

	OPEN	CLOSED
1	A (D CB)	
2	B 15, C 16, D 8	A
3	E 27, F 20, G 17, C 16, D 8	B
4	I 36, J 32, F 20, G 17, C 16, D 8	E
5	J 32, F 20, G 17, C 16, D 8	I
6	G 17, C 16, D 8	J
7	J 20, H 33, C 16, D 8	J

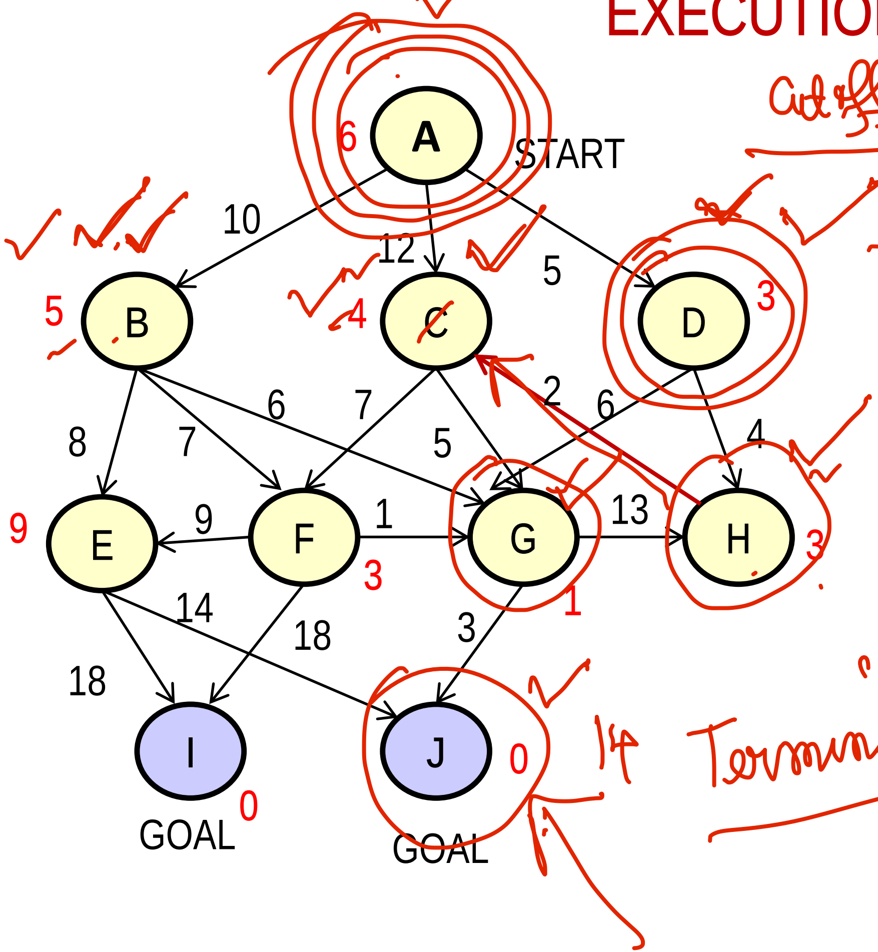
ALGORITHM IDA*



ITERATIVE DEEPENING A* (IDA*)

1. Set Cut-off Bound to f(s)
2. Perform DFBB with Cut-off Bound. Backtrack from any node whose f(n) > Cut-off Bound.
3. If Solution is Found, at the end of one Iteration, Terminate with Solution
4. If Solution is not found in any iteration, then update Cut-off Bound to the lowest f(n) among all nodes from which the algorithm Backtracked.
5. Go to Step 2
6. PROPERTIES OF DFBB AND IDA*: Solution Cost, Memory, Node expansions, Heuristic Accuracy, Performance on Trees / Graphs

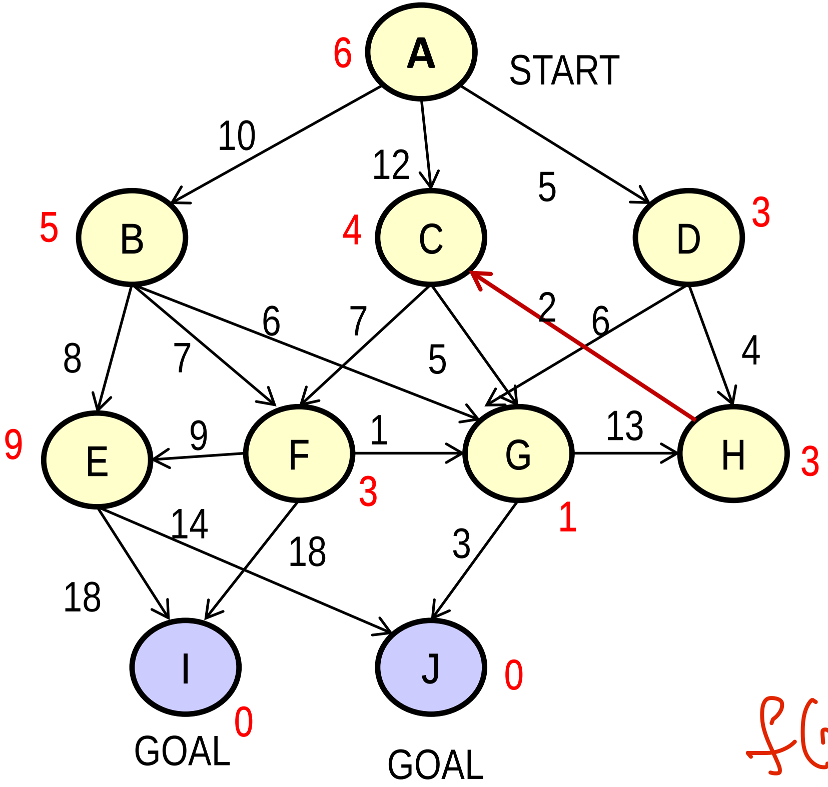
EXECUTION OF IDA*



Cutoff	
6	15, 16, 8
8	15, 16, 12, 12
12	15 15, 14
14	

14 Terminate

COMPARING A*, DFBB & IDA*



1. TERMINATION ISSUES ✓
2. MEMORY AND NODE EXPANSIONS ✓
3. HEURISTIC ACCURACY EFFECT ✓
4. MAXIMIZATION PROBLEMS

Maximization ?

$f(n) = g(n) + h(n)$

$h(n) \Rightarrow$ overestimate

2: Remove the node in OPEN with LARGEST $f(n)$

KNAPSACK PROBLEM: MAXIMIZATION

State configuration

→ items currently chosen, total weight, total value

⟨{ }, 0, 0⟩

taken → ⟨{1}, 10, 1000⟩

not taken → ⟨{ }, 0, 0⟩

State transformation rules: item 1! item 2

h(n)

Trace of A*, DFBB, IDA*

Thank you